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A Note on Partition Functions of Gepner Model Orientifolds

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Abstract

In this note we generalize the description of simple current extended Gepner Model orientifolds as presented in hep-th/0401148 to the case of even levels and non-trivial dressings of the parity transformation. We provide a comprehensive list of all the important ingredients for the construction of such orientifolds. Namely we present explicit expressions for the Klein-bottle, annulus and Möbius strip amplitudes and derive the general tadpole cancellation conditions. As an example we construct a supersymmetric Pati-Salam like model.

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1. Introduction

It is still an open questions whether string theory really contains solutions to its equations of motion resembling the particle physics we observe at low energies. In order to definitely answer this question, we eventually have no other choice than to construct various string backgrounds and check whether Standard Model like features can be achieved. Various classes of four-dimensional string compactifications have been studied in some detail during the last twenty years.

Most recently, there has been extended work on the construction of models using intersecting D-branes as an essential ingredient to get unitary gauge symmetries and chirality [1-7]. Though non-supersymmetric Standard-like models could be found fairly generically [4], an intersecting brane realization of the MSSM using just toroidal orbifold backgrounds turned out to be much harder to achieve [5].

After some earlier studies [8,9], during the last months we have seen a renewed interest in the construction of orientifolds of Gepner models [10-15], which allows one to really move beyond the framework of toroidal orbifolds and to study intersecting brane worlds on small scale Calabi-Yau manifolds [16]. Historically, essentially two approaches have been followed so far. The first one starts on the level of one-loop partition functions and extracts the tadpoles from the explicitly computed Klein-bottle, annulus and Möbius strip amplitudes [8,9,10,11,13,15]. The second approach starts directly on the level of crosscap states in these conformal field theories [17-28,12,14] and then introduces boundary states to cancel the crosscap tadpoles. From there one moves forward to determine the loop-channel amplitudes. Apparently, these two approaches are completely equivalent.

The aim of this note is nothing more than to bring both approaches on equal footing and to relax the assumptions under which the results of [11,13] have been derived using the first approach. More concretely, we generalize the one-loop partition functions, as derived in [11,13] for levels being odd, to the case of even levels. Moreover, on the level of partition functions we implement additional dressings of the word-sheet parity symmetry and identify them with the dressings introduced in [12] in the crosscap state approach. As expected, all the physical information can be read off entirely from the various amplitudes. We will end up with a collection of very explicit and general one-loop partition functions and tadpole cancellation conditions covering simple current extensions of all 168 Gepner models with additional dressings of the parity symmetry. In fact providing a compact collection of the main relevant formulas for constructing supersymmetric Gepner Model

orientifolds was one of the motivations for writing this letter. We hope that these expressions turn out to be useful for a systematic search for standard-like models respectively for providing a statistical ensemble in the spirit of [29].

This paper is organized as follows. In section two we generalize the computation of the A-type loop channel Klein-bottle amplitude to the case of even levels allowing as well certain dressings of the parity symmetry. After deriving the tree-channel amplitude using the methods of [11,13], we determine explicitly the NS-NS sector crosscap state including all sign factors. Section three deals with the open string sector and after computing the Möbius strip amplitude we fix the action of the dressed parity transformation on the boundary states. In section 4 we derive the form of the tadpole cancellation conditions and present a Pati-Salam like model in section 5, providing evidence that phenomenologically interesting Standard-like models are likely to be contained in the huge set of Gepner Model orientifolds [14].

2. Orientifolds of extended Gepner models: The A-type Klein bottle

In [13] we have derived one-loop partition functions for simple current [30] extended Gepner model [31] orientifolds under the assumption of all levels being odd. In this section we repeat the analysis but give up this latter restriction and allow some of the levels to be even. In this case some of the 168 Gepner models with $c = 9$ have only four tensor factors, but, as pointed out in [12], this case should be treated as having five tensor factors with $k_5 = 0$.

For an explanation of the notation to be used in the following and an introduction into Gepner model orientifolds we would like to refer the reader to our former papers [11,13] and references therein. Our starting point here is the simple current extended charge conjugated Gepner model torus partition function

$$\begin{aligned} Z_C(\tau,\bar\tau) = \frac{1}{N}\frac{1}{2^r}\frac{(\mathrm{Im}\tau)^{-2}}{|\eta(q)|^2} \sum_{b_0=0}^{K-1}\sum_{b_1,\ldots,b_r=0}^{1}\sum_{\tau_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\tau_I=0}^{\mathcal{N}_I-1}\sum_{\lambda,\mu}^{\beta}(-1)^{s_0} \\ \prod_{\alpha=1}^{I}\delta^{(1)}\left(Q^{(\alpha)}_{\lambda,-\mu}+2\,\tau_\alpha\hat{Q}^{(\alpha)}(J_\alpha)\right)\,\chi^\lambda_\mu(q)\;\chi^\lambda_{-\mu+b_0\beta_0+b_1\beta_1+\ldots+b_r\beta_r+\sum_\alpha 2\,\tau_\alpha j_\alpha}(\bar q), \end{aligned} \tag{2.1}$$

with $K = \mathrm{lcm}(4, 2k_j{+}4)$ and where we have taken I different mutually local simple currents J_α of length $\mathcal{N}_\alpha$ and where $Q^{(\alpha)}_{\lambda,\mu}$ denotes the monodromy charge of the field (λ,μ) with

respect to the simple current J_α. Let us assume that the simple currents only act on the internal sector, so that they can be brought to the form

$$j_\alpha = (0; m_1^\alpha, \ldots, m_r^\alpha; 0, \ldots, 0) \tag{2.2}$$

with all m_j^α even. As is evident from (2.1), the states surviving the Ω-projection in the charge conjugated (A-type) partition function have to satisfy

$$\mu \cong -\mu + b_0\beta_0 + b_1\beta_1 + \ldots + b_r\beta_r + \sum_\alpha \tau_\alpha j_\alpha, \tag{2.3}$$

i.e.

$$\begin{aligned} m_j &= b + \sum_\alpha \tau_\alpha \frac{m_j^\alpha}{2} + \frac{1}{2}\eta_j(k_j+2) \mod (k_j+2) \quad \text{for all } j, \\ s_0 &= b + \sum_i b_i \mod 2, \\ s_j &= b + b_j + \eta_j \mod 2 \end{aligned} \tag{2.4}$$

for some b in the range $\{0, \ldots, \frac{K}{2} - 1\}$, $b_j = 0, 1$. The only change compared to the case of only odd levels is the appearance of η_j, which takes the values $\eta_j = 0, 1$ in every tensor factor where $l_j = \frac{k_j}{2}$ and vanishes otherwise. Therefore it is only present for even $K' = \mathrm{lcm}(k_j + 2)$. The origin of η_j is due to the fact that for even levels the value $l_j = \frac{k_j}{2}$ is invariant under the reflection symmetry $(l_j, m_j, s_j) \to (k_j - l_j, m_j + k_j + 2, s_j + 2)$, thus leading to the existence of shorter simple current orbits. The constraints on s_j and s_0 imply

$$\sum_j \eta_j = 0 \mod 2. \tag{2.5}$$

Since our aim is to exploit the resulting expressions for a systematic examination of the spectrum, it turns out to be useful to require that for all pairs of simple currents $Q^{(\alpha)}(J_\beta) = \sum_j (m_j^\alpha m_j^\beta)/(2k_j + 4)$, is an even integer. This will simplify the calculations and the resulting expressions considerably. These projections are then implemented as in [13]. As is well known, however, the orientifold projection is by no means unique in the sense that one is always free to dress the characters which survive the projection with additional signs consistent with the fusion rules [32].

In view of the free parameters in (2.3) and the various relations (2.4) between them, we define the orientifold projection $\Omega_{\Delta_j,\omega,\omega_\alpha}$ by including the sign factors

$$(-1)^{\omega\,(b+s_0) + \sum_j \Delta_j \eta_j + \sum_\alpha \omega_\alpha\,\tau_\alpha} \tag{2.6}$$

for $\Delta_i, \omega, \omega_\alpha = 0, 1$. Note that the Δ_j only have a non-trivial effect if k_j is even. Moreover, the combination $(b + s_0)$ is just right for the ω dressing to preserve supersymmetry of the resulting Klein-bottle amplitude and is only well defined for K' even. Similarly, a non-trivial simple current dressing, $\omega_\alpha = 1$, is only allowed for $\mathcal{N}_\alpha$ even. Independently of these optional parity dressings, consistency with our results from [13] for the case of all levels being odd requires a factor of $\prod_{k<l}(-1)^{\eta_k \eta_l}$. Then, the overall A-type Klein bottle can be written as

$$
\begin{aligned}
K^A(\Delta_j, \omega, \omega_\alpha) = 4 \int_0^\infty \frac{dt}{t^3} \frac{1}{2^{r+1}} \frac{1}{\eta(2it)^2} \sum_{\lambda,\mu}^{\beta} \sum_{\eta_1,\ldots,\eta_r=0}^{1} \sum_{b=0}^{\frac{K}{2}-1} \sum_{\tau_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\tau_I=0}^{\mathcal{N}_I-1} (-1)^{s_0} \, (-1)^{\omega(b+s_0)} \\
(-1)^{\sum_j \Delta_j \eta_j} \, (-1)^{\sum_\alpha \omega_\alpha \tau_\alpha} \, \delta^{(2)}_{\sum_j \eta_j, 0} \, \left(\prod_{k<l} (-1)^{\eta_k \eta_l} \right) \left(\prod_j \delta_{l_j \eta_j, \frac{k_j}{2} \eta_j} \right) \\
\left(\prod_\alpha \delta^{(1)}_{\sum_j \frac{1}{4} \eta_j m_j^\alpha, 0} \right) \left(\prod_{j=1}^{r} \delta^{(k_j+2)}_{m_j, b + \sum_\alpha \frac{1}{2} \tau_\alpha m_j^\alpha + \eta_j \frac{1}{2}(k_j+2)} \right) \chi^\lambda_\mu(2it),
\end{aligned}
\tag{2.7}
$$

where the first term in the last line is a remnant of the monodromy charge constraint in (2.1). The tree-channel amplitude is modified accordingly as

$$
\begin{aligned}
\widetilde{K}^A(\Delta_j, \omega, \omega_\alpha) = \frac{2^4 \prod_\alpha \mathcal{N}_\alpha}{2^{\frac{3r}{2}} \prod_j \sqrt{k_j+2}} \int_0^\infty dl \frac{1}{\eta^2(2il)} \sum_{\lambda',\mu'}^{ev} \sum_{\eta_1,\ldots,\eta_r=0}^{1} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \\
\left(\prod_{k<l} (-1)^{\eta_k \eta_l} \right) \left(\prod_\alpha \delta^{(1)}_{\sum_j \frac{1}{4} \eta_j m_j^\alpha, 0} \right) \left(\prod_\alpha \delta^{(2)}_{Q^{(\alpha)}_{\lambda', \mu' + (1-\vec{\epsilon})(\vec{k}+2)}, \omega_\alpha} \right) \\
\delta^{(2)}_{\sum_j \eta_j, 0} \; \delta^{(4)}_{s_0' + \nu_0 + 2\sum \nu_j + 2, 2\omega} \; \delta^{(2)}_{\sum_j \frac{1}{k_j+2}(m_j' + (1-\epsilon_j)(k_j+2)), \omega} \\
\prod_{j=1}^{r} \left(\frac{P_{l_j', \epsilon_j k_j} P_{l_j', (\epsilon_j+\eta_j) k_j}}{S_{l_j', 0}} \; \delta^{(2)}_{\eta_j k_j, 0} \; (-1)^{\eta_j \left(\frac{m_j'}{2} + \nu_0 + \Delta_j + (1-\epsilon_j) \right)} \right. \\
\left. \delta^{(2)}_{m_j' + (1-\epsilon_j)(k_j+2), 0} \; \delta^{(4)}_{s_j' + \nu_0 + 2\nu_j + 2(1-\epsilon_j), 0} \right) \chi^{\lambda'}_{\mu'}(2il),
\end{aligned}
\tag{2.8}
$$

where we have introduced the short-hand notation $(1 - \vec{\epsilon})(\vec{k} + 2)$ for the vector $\mu_{\vec{\epsilon}} = (0; (1-\epsilon_1)(k_1+2), \ldots, (1-\epsilon_5)(k_5+2); 0, \ldots, 0)$. Note that besides the appearance of the sum over the parameters η_j also the conditions on the monodromy charges with respect to the additional simple currents changes slightly as compared to the case of all levels being odd.

From the tree-channel Klein bottle, we can read off the crosscap state up to overall signs and complex phases which cancel in the overlap. These phases fall into two classes: Those depending only on the states contributing to the crosscap and those which are a function of the parameters of the dressings. For the determination of the signs in the first class we follow the method presented in [11,13] (which was shown to work in the NS-NS sector and is therefore sufficient for supersymmetric models). The second class of phases has no physical meaning as they can be rotated away. Once a particular choice is made, however, it determines the parity action on the boundary states uniquely, as we will see from the Möbius amplitude.

For pure convenience, we choose to include the phase factor

$$(-1)^{\omega \frac{s_0'}{2}} e^{i\pi \sum_j \frac{\Delta_j\, (m_j'+k_j+2)}{k_j+2}} \tag{2.9}$$

into the crosscap state. Independently of this convention, an additional $\exp(i\pi\sum_j \Delta_j\epsilon_j)$ is really required to obtain (2.8) correctly, so that the final crosscap state takes the form

$$\begin{aligned}
\big|C;\Delta_j,\omega,\omega_\alpha\big\rangle_{NS} = \frac{1}{\kappa_c^A} \sum_{\lambda',\mu'}^{ev} \sum_{\nu_0=0}^{\frac{K}{2}-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} & (-1)^{\nu_0} \left(\prod_{k<l}(-1)^{\nu_k\nu_l}\right) (-1)^{\sum_j \nu_j} \\
& (-1)^{\omega\frac{s_0'}{2}}\, e^{i\pi\sum_j \frac{\Delta_j}{k_j+2}(m_j'+(1-\epsilon_j)(k_j+2))} \left(\prod_\alpha \delta^{(2)}_{Q^{(\alpha)}_{\lambda',\mu'+(1-\vec{\epsilon})(\vec{k}+2)},\omega_\alpha}\right) \\
& \delta^{(4)}_{s_0'+2\nu_0+2\sum\nu_j+2,2\omega}\, \delta^{(2)}_{\sum_j \frac{1}{k_j+2}(m_j'+(1-\epsilon_j)(k_j+2)),\omega} \prod_{j=1}^{r}\Bigg(\sigma(l_j',m_j',s_j')\frac{P_{l_j',\epsilon_j k_j}}{\sqrt{S_{l_j',0}}} \\
& (-1)^{\epsilon_j\frac{m_j'+s_j'}{2}}\, \delta^{(2)}_{m_j'+(1-\epsilon_j)(k_j+2),0}\, \delta^{(4)}_{s_j'+2\nu_0+2\nu_j+2(1-\epsilon_j),0}\Bigg)\ \big|\lambda',\mu'\big\rangle\!\big\rangle_c,
\end{aligned} \tag{2.10}$$

where

$$\left(\frac{1}{\kappa_c^A}\right)^2 = \frac{2^5\left(\prod_{\alpha=1}^I \mathcal{N}_\alpha\right)}{2^{\frac{3r}{2}} K \prod_j \sqrt{k_j+2}}. \tag{2.11}$$

Comparing this crosscap state for $\omega_\alpha = 0$ to the one used in [12] one finds complete agreement. Therefore we conclude that the Δ_j really define the various phase dressings and ω the quantum dressing of the parity transformation as introduced in [12]. Note that the crosscap state (2.10) in addition includes the non-trivial simple current dressings ω_α.

As anticipated before, the form of the crosscap state for the case of even levels does not differ at all from its analogue for the case of all levels being odd. The new parameters, η_j, in the former case simply arise from additional contributions in the overlap of the crosscap state with itself and therefore arise automatically from (2.10).

3. Open string one loop amplitudes

As usual, in order to cancel the massless tadpoles of the orientifold planes one introduces A-type boundary states, which for a simple current extension have the form

$$\begin{aligned} |a\rangle_A = \left|S_0; (L_j, M_j, S_j)_{j=1}^r\right\rangle_A = \frac{1}{\kappa_a^A} \sum_{\lambda',\mu'}^{\beta} \prod_\alpha \delta^{(1)}\big(Q^{(\alpha)}_{\lambda',\mu'}\big)(-1)^{\frac{s_0'^2}{2}} e^{-i\pi \frac{s_0' S_0}{2}} \\ \prod_{j=1}^r \left(\frac{S_{l_j',L_j}}{\sqrt{S_{l_j',0}}}\, e^{i\pi \frac{m_j' M_j}{k_j+2}}\, e^{-i\pi \frac{s_j' S_j}{2}} \right) |\lambda', \mu'\rangle\rangle \end{aligned} \tag{3.1}$$

with the normalization

$$\frac{1}{\left(\kappa_a^A\right)^2} = \frac{K\ \left(\prod_\alpha \mathcal{N}_\alpha\right)}{2^{\frac{r}{2}+1} \prod_j \sqrt{k_j+2}}. \tag{3.2}$$

Note that boundary state labels connected by the action of the simple currents J_α describe identical D-branes. In order to finally read off the massless spectrum, we have to transform their overlap into loop channel

$$\begin{aligned} A^A_{\tilde{a}\,a} = N_a N_{\tilde a}\, \frac{1}{2^{r+1}} \int_0^\infty \frac{dt}{t^3} \frac{1}{\eta^2(it)} \sum_{\lambda,\mu}^{ev} \sum_{\nu_0=0}^{K-1} \sum_{\nu_1,\ldots,\nu_r=0}^{1} \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} \sum_{\sigma_1=0}^{\mathcal{N}_1-1} \cdots \sum_{\sigma_I=0}^{\mathcal{N}_I-1} (-1)^{\nu_0} \\ \delta^{(4)}_{s_0, 2+\tilde S_0 - S_0 - \nu_0 - 2\sum_j \nu_j} \prod_{j=1}^r \Bigg(N^{|\epsilon_j k_j - l_j|}_{L_j, \tilde L_j}\ \delta^{(2k_j+4)}_{m_j + M_j - \tilde M_j + \nu_0 + \sum_\alpha \sigma_\alpha m_j^\alpha + \epsilon_j (k_j+2), 0} \\ \delta^{(4)}_{s_j, \tilde S_j - S_j - \nu_0 - 2\nu_j + 2\epsilon_j} \Bigg) \chi^\lambda_\mu(it). \end{aligned} \tag{3.3}$$

It is well known that for even levels some of the boundary states (3.1) are not fundamental and split into fractional branes. These so-called resolved boundary states have been constructed in [33,34]. Here just for keeping the presentation simple we work with the unresolved Recknagel/Schomerus [35,36] boundary states (3.1).

Let us now address the issue of the action of $\Omega_{\Delta_j,\omega,\omega_\alpha}$ on a boundary state. For this purpose, we compute the overlap of a boundary state with the crosscap state (2.10), which by the way is not different for the resolved boundary states, as the crosscap state only

contains untwisted contributions. After transforming into loop channel we obtain

$$\begin{aligned}
M_a^{A,NS}(\Delta_j,\omega,\omega_\alpha) &= (-1)^s N_a\ \frac{1}{2^{r+1}}\int_0^\infty \frac{dt}{t^3}\,\frac{1}{\hat\eta^2(it+\frac{1}{2})}\sum_{\lambda,\mu}^{ev}\ \sum_{\nu_0=0}^{\frac{K}{2}-1}\ \sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1}\ \sum_{\sigma_1=0}^{\mathcal{N}_1-1}\cdots\sum_{\sigma_I=0}^{\mathcal{N}_I-1} \\
&(-1)^{\omega(\nu_o+\frac{s_0}{2})}\ (-1)^{\sum_\alpha \omega_\alpha\tau_\alpha}\ \left(\prod_{k<l}(-1)^{\rho_k\rho_l}\right)\ \delta^{(2)}_{\sum_j \rho_j,0}\ \delta^{(2)}_{s_0,0} \\
&\prod_{j=1}^r\Bigg(\sigma_{(l_j,m_j,s_j)}\,Y^{l_j}_{L_j,\epsilon_j k_j}\ \delta^{(2)}_{s_j,0}\ \delta^{(2k_j+4)}_{2(M_j-\Delta_j)+m_j+2\nu_0+\sum_\alpha\sigma_\alpha m_j^\alpha+\epsilon_j(k_j+2),0} \\
&(-1)^{\frac{\epsilon_j}{2}[2S_j-s_j-2\epsilon_j]}(-1)^{\frac{(1-\epsilon_j)}{2}[2M_j-m_j+\epsilon_j(k_j+2)]}\Bigg)\ \hat\chi^\lambda_\mu(it+\frac{1}{2}),
\end{aligned}
\tag{3.4}$$

where

$$\begin{aligned}
r &= 4s+1, \\
\rho_j &= \frac{s_0+s_j}{2}+\omega+\epsilon_j-1,
\end{aligned}
\tag{3.5}$$

and the Y-tensor is defined as

$$Y^{l_3}_{l_1,l_2} = \sum_{l=0}^{k}\frac{S_{l_1,l}\,P_{l_2,l}\,P_{l_3,l}}{S_{0,l}}. \tag{3.6}$$

Requiring that the Möbius amplitude (3.4) is consistent with the annulus amplitude (3.3) for a D-brane and its $\Omega_{\Delta_j,\omega,\omega_\alpha}$ image, we can derive the action of $\Omega_{\Delta_j,\omega,\omega_\alpha}$ on a boundary state. First note that Ω itself reverses the sign of the labels S_0, M_j, S_j. The phase dressings shift the M_j to $M_j+2\Delta_j$ and the ω dressing changes the GSO projection in (3.4) and therefore maps a brane to its anti-brane, which can also be described by the shift $S_0\to S_0+2$. Finally, the ω_α dressings only change some sign factors in (3.4) and therefore should leave a boundary state invariant. To summarize, the entire action of $\Omega_{\Delta_j,\omega,\omega_\alpha}$ on a boundary state is given by

$$\big|S_0,\prod_j(L_j,M_j,S_j)\big\rangle\big\rangle \to \big|-S_0+2\omega,\prod_j(L_j,-M_j+2\Delta_j,-S_j)\big\rangle\big\rangle. \tag{3.7}$$

In particular, the invariant branes of the pure non-extended Gepner Model are now classified by

$$|S_0,\prod_{j=1}^{r'}(\frac{k_j}{2},\Delta_j+\frac{k_j+2}{2},S_j)\ \prod_{j=r'+1}^{r}(L_j,\Delta_j,S_j)\rangle\rangle \tag{3.8}$$

for $(r'-\omega)$ even and the M_j chosen modulo (k_j+2). A boundary state is supersymmetric relative to the crosscap state if

$$\frac{S_0-\omega}{2}-\sum_j \frac{M_j-\Delta_j}{k_j+2}+\sum_j \frac{S_j}{2}=0 \bmod 2. \tag{3.9}$$

From these latter expressions it is clear that the phase dressings can be thought of as a rotation in the M_j planes, whereas the quantum ω dressing similarly can be considered as a rotation in S_0 plane. Therefore, from the conformal field theory point of view the phase shifts and the quantum dressing are completely analogous.

4. Tadpole cancellation conditions

The tadpole cancellation conditions contain both the contribution from the D-branes and from the orientifold planes and take the general form $\mathrm{Tad}_D(\lambda,\mu)-4\mathrm{Tad}_O(\lambda,\mu)=0$ for the massless fields $(2)(0,0,0)^5$ and $(0)\prod_j(l_j,l_j,0)$ with $\sum_j \frac{l_j}{k_j+2}=1$. Up to the common factor

$$\text{const.}\times \frac{e^{i\pi\sum_j \frac{\Delta_j}{k_j+2}m_j}}{\prod_j \sqrt{S_{l_j,0}}}, \tag{4.1}$$

containing in particular the phase convention mentioned in the end of section 2, the NS-NS tadpoles of the orientifold plane read

$$\begin{aligned}\mathrm{Tad}_O(\lambda,\mu)=(-1)^{(1+\frac{s_0}{2})(1+\omega)} &\sum_{\epsilon_1,\ldots,\epsilon_r=0}^{1} e^{i\pi\sum_j \frac{\Delta_j}{k_j+2}(1-\epsilon_j)(k_j+2)}\left(\prod_{k<l}(-1)^{\epsilon_k\epsilon_l}\right)\\ &\delta^{(2)}_{\sum\epsilon_j,\omega+\frac{s_0}{2}}\left(\prod_\alpha \delta^{(2)}_{Q^{(\alpha)}_{\lambda,\mu+(1-\vec\epsilon)(\vec k+2)},\omega_\alpha}\right)\prod_j\Bigg(\sin\left[\frac{1}{2}(l_j,\epsilon_j k_j)\right]\ \delta^{(2)}_{l_j+(1-\epsilon_j)k_j,0}\\ &\delta^{(2)}_{m_j+(1-\epsilon_j)(k_j+2),0}\ (-1)^{\epsilon_j\frac{m_j}{2}}\Bigg).\end{aligned} \tag{4.2}$$

Note that for k_j even only those massless states with m_j even do have a non-vanishing tadpole on the orientifold plane. Collecting all terms from the boundary states and their $\Omega_{\Delta_j,\omega,\omega_\alpha}$ images, their massless tadpoles read

$$\mathrm{Tad}_D(\lambda,\mu)=\left(\prod_\alpha \delta^{(1)}_{Q^{(\alpha)}_{\lambda,\mu}}\right)\sum_{a=1}^{N} 2\,N_a\cos\left[\pi\sum_j \frac{m_j(M^a_j-\Delta_j)}{k_j+2}\right]\prod_j \sin(l_j,L^a_j). \tag{4.3}$$

By now we have provided a comprehensive collection of the salient formulas needed to construct orientifolds of Gepner Models. We featured all one-loop partition functions and the resulting tadpole cancellation conditions covering simple current extended Gepner model orientifolds with generally dressed $\Omega_{\Delta_j,\omega,\omega_\alpha}$ parity. We hope that these very explicit expressions will be helpful for future work on classifying semi-realistic models respectively on carrying out a statistical analysis in the spirit of [29]. As a simple example showing that semi-realistic models are possible to get we present in the final section a two generation supersymmetric Pati-Salam model.

5. A Pati-Salam like example

We take the $(6)^4$ Gepner model, which has Hodge numbers $(h_{21}, h_{11}) = (1, 149)$ but leads after extending it by the two simple currents

$$J_1 = (0; 2, -2, 0, 0, 0; 0, 0, 0, 0, 0), \qquad J_2 = (0; 2, 2, -4, 0, 0; 0, 0, 0, 0, 0) \tag{5.1}$$

to a model with Hodge numbers $(h_{21}, h_{11}) = (69, 5)$. We choose trivial dressing ($\Delta_j = 0, \omega = 0, \omega_\alpha = 0$) and introduce four D-branes of type

$$|S_0^a; \prod_j (L_j^a, M_j^a, S_j^a)\rangle = |0; (1, -7, 0)(0, -6, 0)(3, -7, 0)(0, -4, 0)(0, 2, 0)\rangle \tag{5.2}$$

and their Ω images. From the annulus and Möbius strip amplitude we learn that this brane does not need to be resolved and that it gives rise to a $U(4)$ gauge symmetry. Next we introduce stacks of two D-branes of type

$$|S_0^b; \prod_j (L_j^b, M_j^b, S_j^b)\rangle = |0; (0, -6, 0)(0, -6, 0)(3, -7, 0)(3, -5, 0)(0, 2, 0)\rangle \tag{5.3}$$

and

$$|S_0^c; \prod_j (L_j^c, M_j^c, S_j^c)\rangle = |0; (0, -6, 0)(0, -4, 0)(3, -7, 0)(3, -7, 0)(0, 2, 0)\rangle. \tag{5.4}$$

These D-branes turn out to be not single objects but are made of two fractional branes each. Moreover, each one gives rise to a gauge symmetry $SP(2) \times SP(2) \simeq SU(2) \times SU(2)$. One can show that all six tadpoles do vanish for this configuration and that the intersection numbers give rise to the chiral spectrum as shown in Table 1

deg.	$U(4)\times SU(2)\times SU(2)\times SU(2)\times SU(2)$
2	$(\mathbf{4},\mathbf{2},1,1,1)$
2	$(\mathbf{4},1,\mathbf{2},1,1)$
2	$(\overline{\mathbf{4}},1,1,\mathbf{2},1)$
2	$(\overline{\mathbf{4}},1,1,1,\mathbf{2})$

Table 1: *massless chiral matter spectrum*

Therefore this Gepner model orientifold gives rise to a two generation supersymmetric PS-like model. It is beyond the scope of this paper to dwell upon the phenomenological features of this model. We consider this merely as a hint that supersymmetric Standard-like models are likely to be contained in the enormously huge class of Gepner model orientifolds [1].

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[1] Recently, based on an extensive computer search the authors of [14] claimed to have found three generation Standard-like models.

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